



# Algebraic Reasoning Content Academy – Grade 4

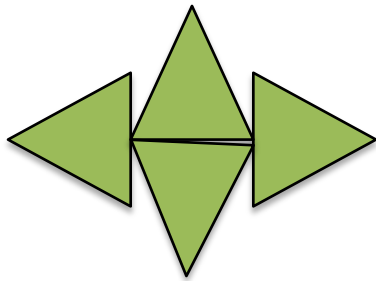
Day 2: Multiple Representations  
Sami Briceño

# Day 2 Learning Outcomes

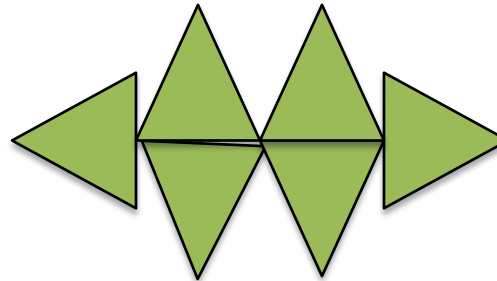
- Use both real-life and abstract representations to solve problems
- Introduce the value of multiple representations: table, picture, equation, graph
- Recognize that constant rate of change is a defining characteristic of linear functions
- Investigate what can be learned from analyzing students' work
- Solve, analyze, and discuss 4th grade algebraic reasoning tasks, connection to MS & HS Algebra Tasks

# Constructing Worms!!

Using pattern blocks, construct the following worms shown below.

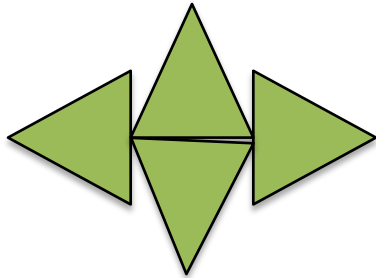


One Day Old Worm

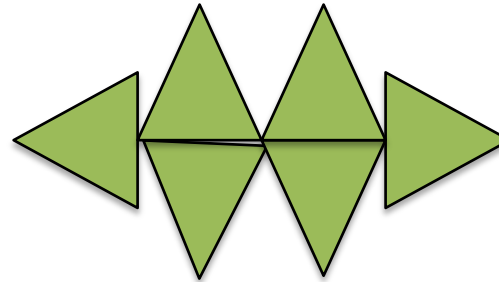


Two Day Old Worm

# Constructing Worms!!



One Day Old Worm



Two Day Old Worm

- 1) How many triangles will be needed to make the next worm?
- 2) Write a rule representing the relationship between the age of the worm,  $w$ , and the number of triangles needed to make the worm,  $n$ .

# Tile Patterns



# Tiling around a Fountain

- **SCENARIO:** You are working on a landscaping crew for the county parks department this summer.
- To avoid the mud that surrounded the park fountains last summer, your crew is planning to put a border of tiles around each of the square fountains in the park. The border tiles each measure 1-foot on each side. Your foreman shows you this diagram for the smallest fountain. You notice that a fountain that has a base of 1 square foot will require 8 border tiles.
- Using this pattern, how many tiles will be needed for different size square fountains?





# Tiling around a Fountain

**Problem 1:** If a square fountain has sides of length  $s$  feet, how many tiles are needed to form the border?

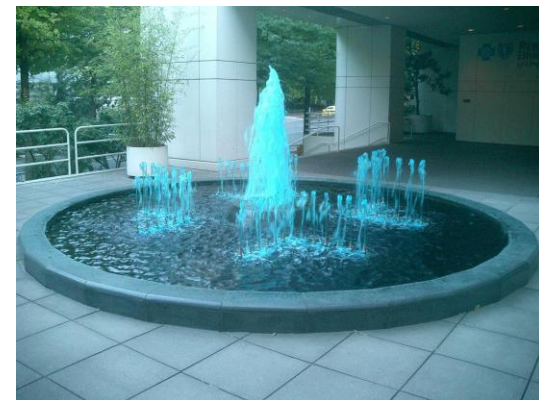
- Using grid paper draw a diagram of the designs for the border of fountains with side lengths of 2, 3, 4, 6 and 10 feet. Record your results in a table.
- Write an equation for the number of tiles,  $N$ , needed to form a border for a square fountain with side length of  $S$  feet.
- Generate as many equations as you can for this relationship.
  - Are the equations the same?
  - How can you convince someone that your expressions for the number of tiles needed are equivalent?



# Group Poster

Tiling around a Fountain

- Show all the expressions generated by your group
- Explicitly connect your expressions to the diagrams/concrete model





# Gallery Walk

## Tiling around a Fountain

- One person from each group “mans” the group’s poster to answer questions.
- Rest of the group members view other posters. Look for:
  - The most common representations
  - The most unique representations



# Re-Tiling

Tiling around a Fountain

**Follow-up Problem:** Make a table and a graph for each equation you found in problem 1.

- Do the tables and graphs show that the expressions and equations are equivalent?
- Is this relationship between  $s$  and  $n$  a straight line, or a curve?



# Collection of Expressions

Tiling around a Fountain

- Make a collective list of all the expressions that were generated in the groups.



# Equivalent Expressions

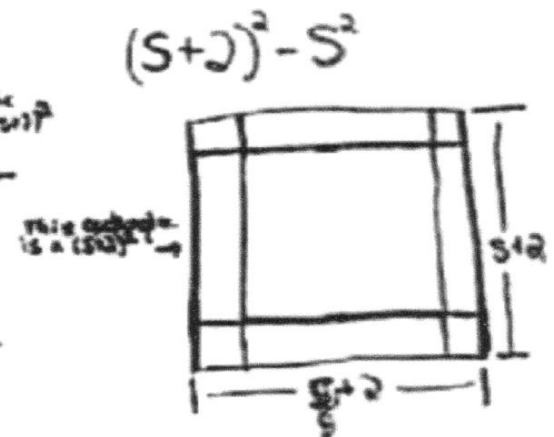
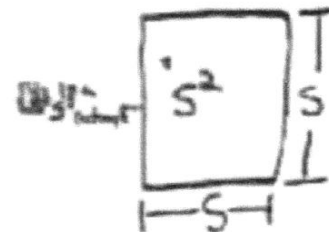
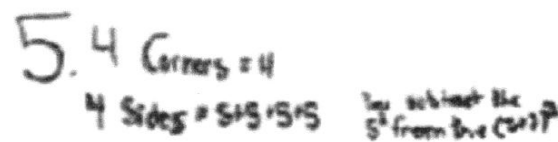
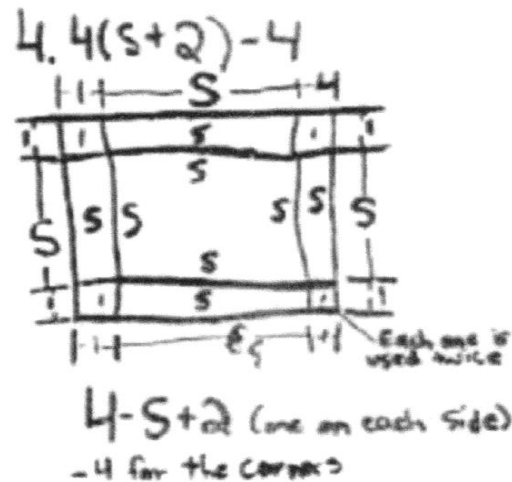
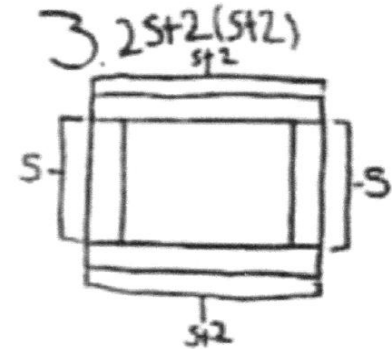
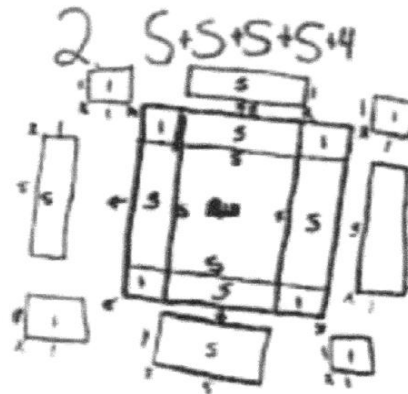
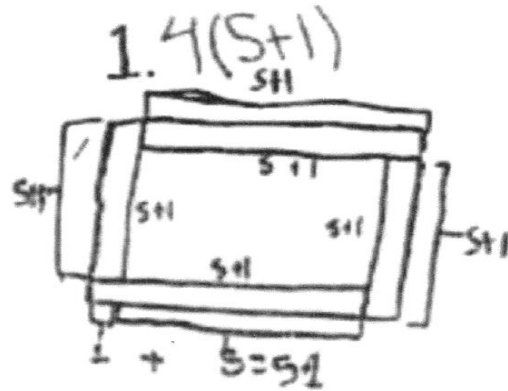
Tiling around a Fountain

Examine the collection of expressions for the tiling problem.

- What are the advantages of each expression?
- Which method was easier to compare expressions for equivalency: tables, equations, or graphs?

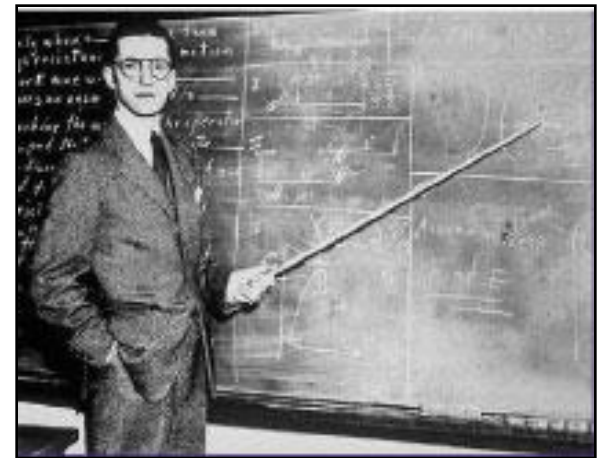


# Tiling around a Fountain



# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?

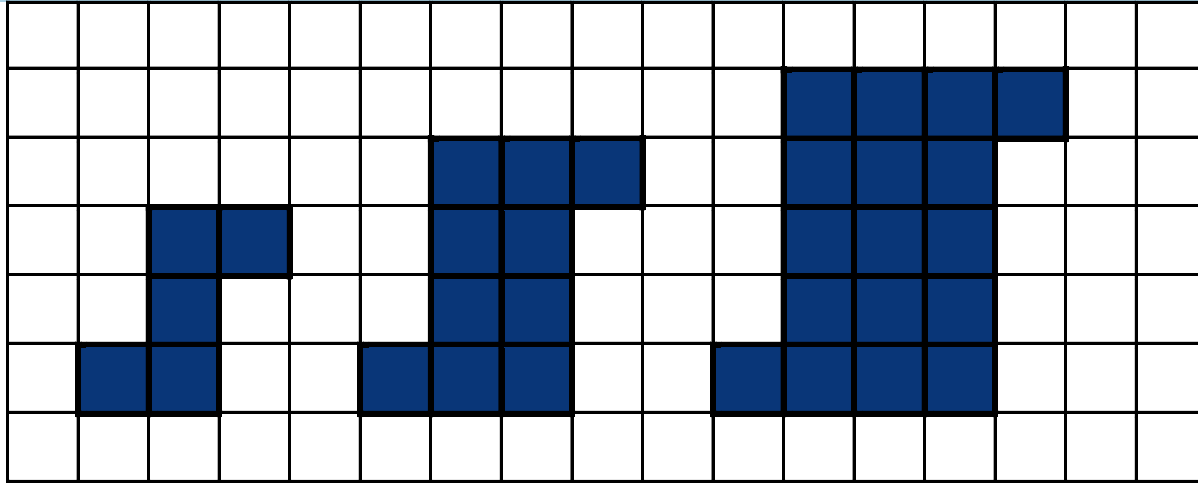




# A Different Tiling Pattern



# The S-Pattern Task



- Find the number of tiles for the next two figures in the pattern.
- Write as many different equations as you can for the number of tiles needed for any figure in the pattern.

# The S-Pattern Task

- Make a graph that shows the relationship between the figure number and the number of tiles in the figure.
- Share your solutions with your group.



# A Graph of the Pattern

## The S-Pattern Task

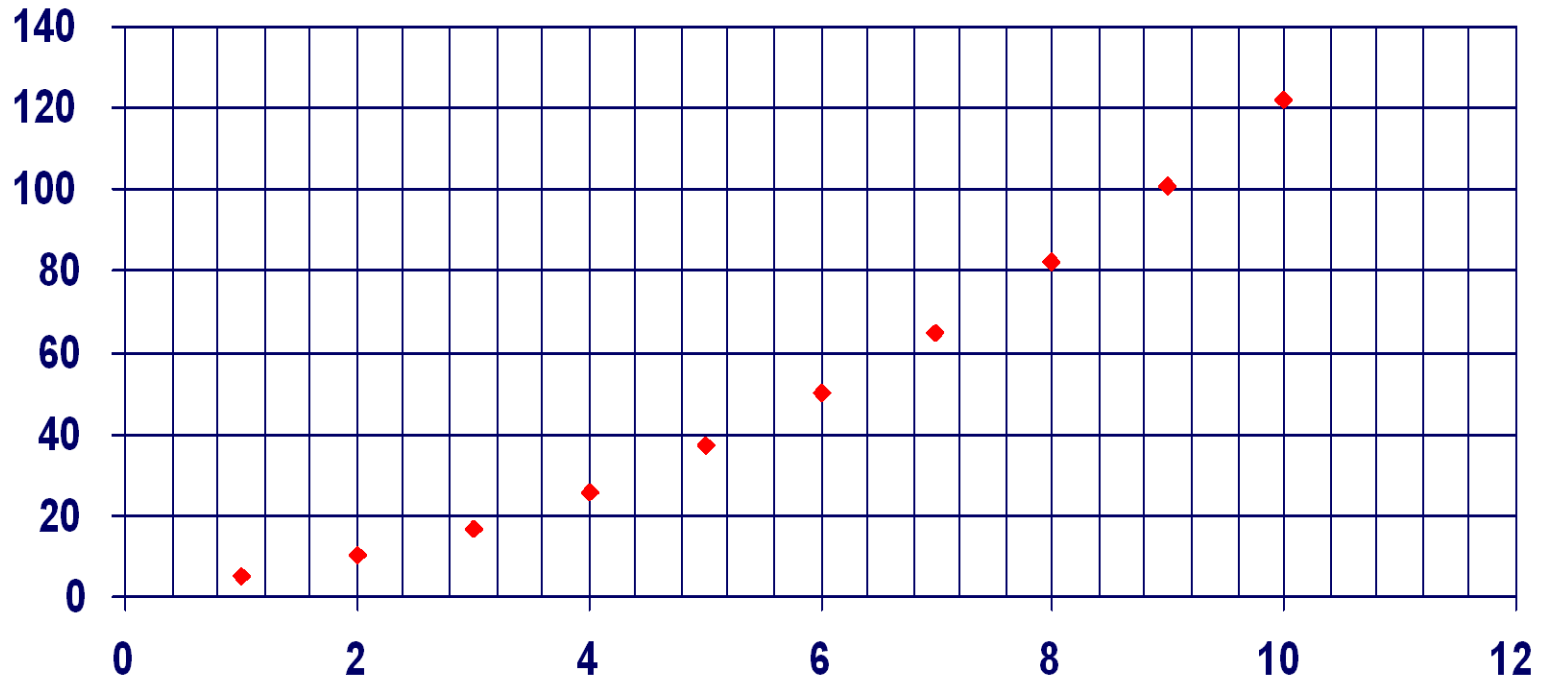
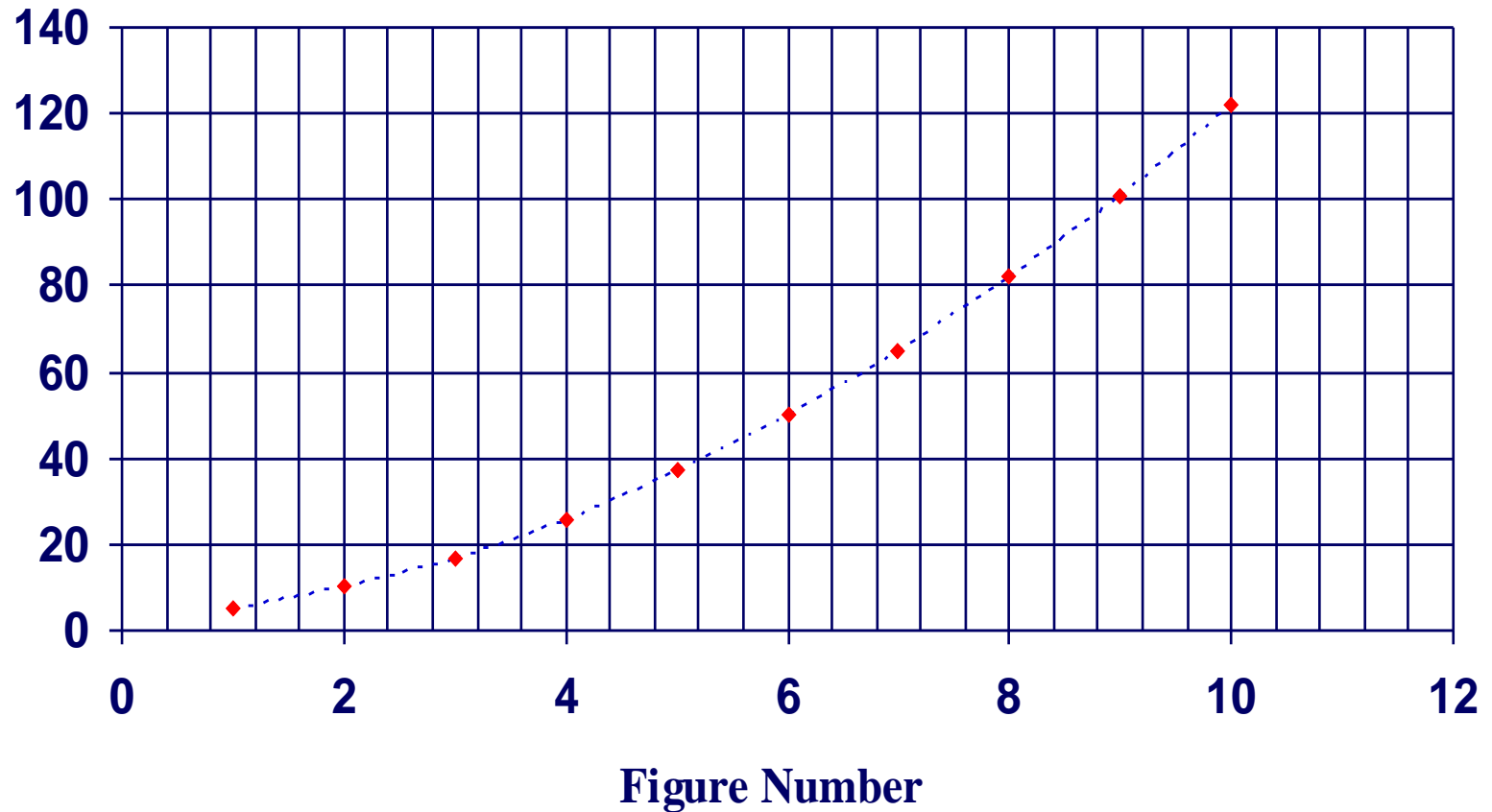


Figure Number

How does the S-Pattern graph compare to the Tiling the Fountain graph?

# Another Graph of the Pattern

The S-Pattern Task



# A Graph of All Values

## The S-Pattern Task

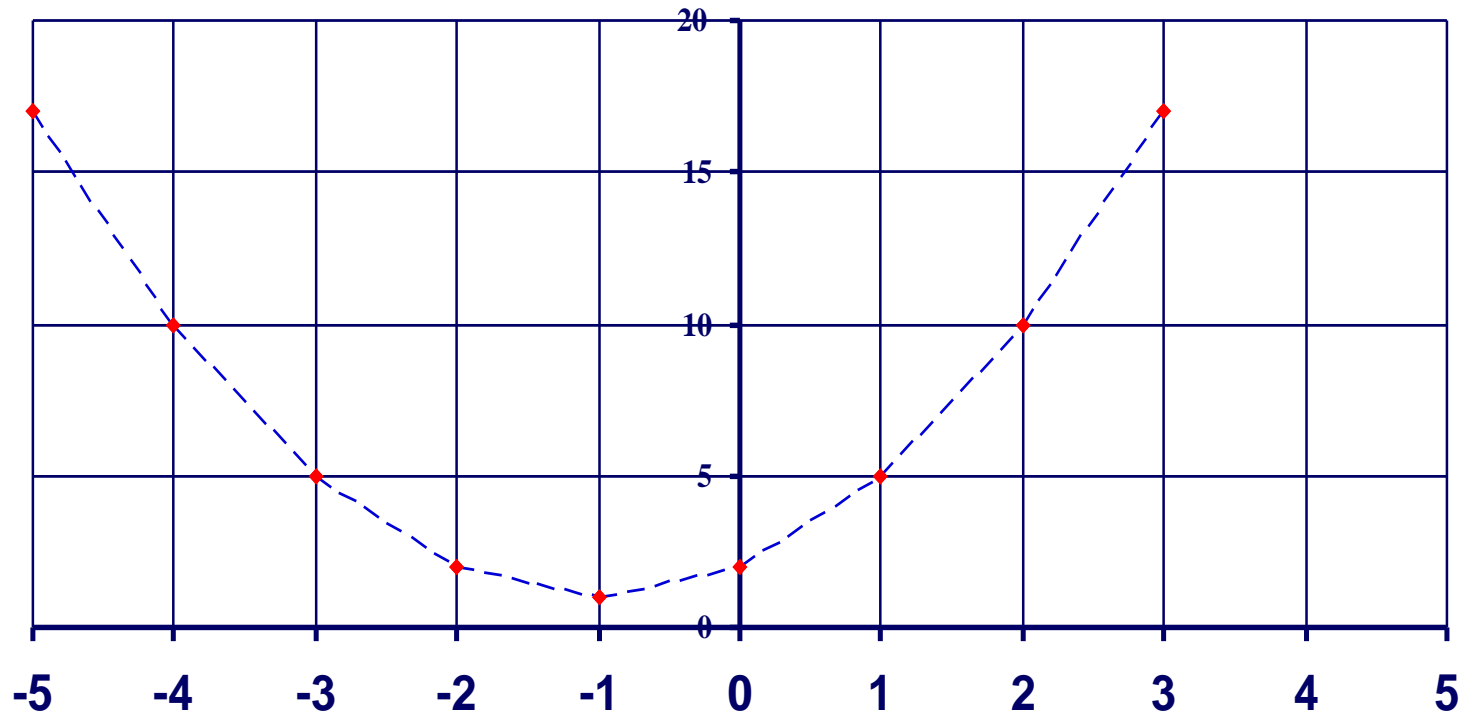


Figure Number



# De-Briefing the S-Pattern Task

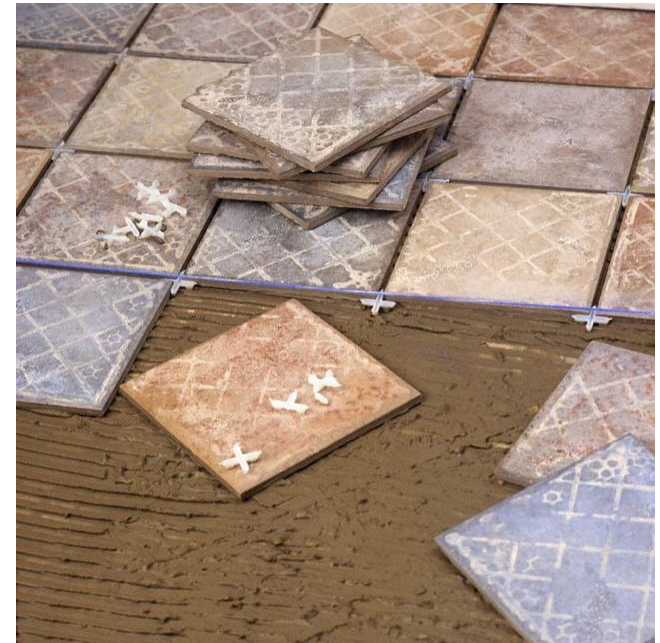
- Contrast the strategies that were presented.

What were the similarities and what were the differences?



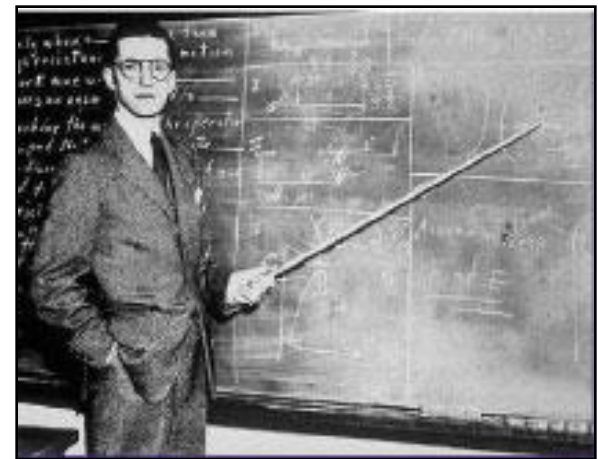
# Comparing the Two Tasks

- How are the patterns in tiling and S-pattern the same?
- How are the patterns different?



# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
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# Why do we want kids to experience Pattern Tasks?

- Understanding variables
- Connecting between representations
- Progressing on standards
- Using context for working on functions
- Starting the problem at entry points
- Finding and comparing solutions



# Connecting Representations

The learner who can, for a particular mathematical problem, move fluidly among different representations has access to a perspective on the mathematics in the problem that is greater than the perspective any one representation can provide.



# Connecting Representations

The nature of the instruction that students receive, in both the representations that are emphasized and the kinds of translation tasks that are presented, may significantly contribute to the difficulties that many students have in connecting equations and graphs.

Knuth,2000





# Connecting Representations

An important aspect of developing a robust understanding of functions means not only knowing which representation is most appropriate for use in different contexts but also being able to move flexibly between different representations in different directions.



# Linking to Research/Literature

As students work with multiple representations of functions - **including numeric, graphic, and symbolic** - they will develop a more comprehensive understanding of functions.

NCTM, 2000, p. 38



# Analyzing Student Work

## The S-Pattern Task

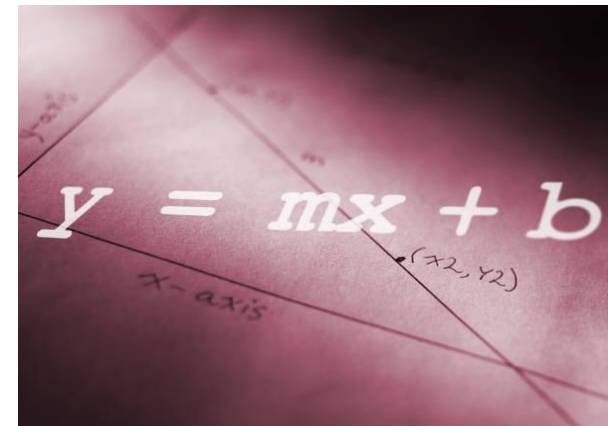
- Examine each students' response.  
What does it suggest the student understands?  
Does not understand? Why?
  - Which response do you think shows the greatest understanding? Why?
  - Which response do you think shows the least understanding? Why?
- Record your group's strongest and weakest response on the class recording sheet.

# Analyzing Student Work

## The S-Pattern Task

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

# Examining Linear Functions



# Roger's Roofing

Roger's Roofing Company was asked to put shingles on many new garages. The company decided to start with the smallest garage and continue until they were putting shingles on large business garages. The first garage took 3 loads of shingles. The second garage took 5 loads of shingles, and the third garage took 7 loads of shingles. The fourth garage took 9 loads of shingles.





# Roger's Roofing

- How many loads of shingles will the fifth garage use?
- How many loads of shingles will the fourteenth garage use?
- How many loads of shingles will the thirty-eighth garage use?



# Roger's Roofing

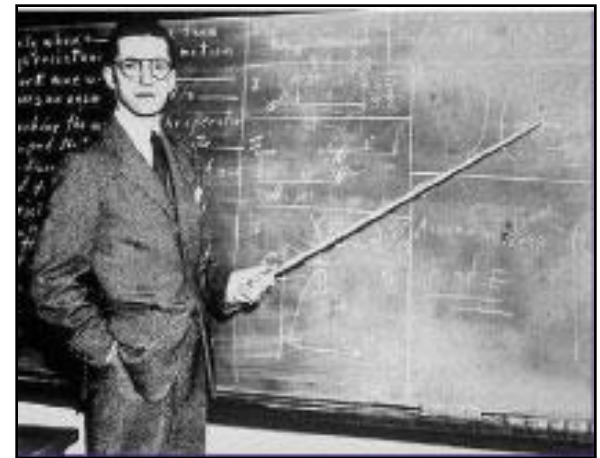
Each load of shingles cost \$120.00. The business that owns the nineteenth garage has budgeted \$5,500.00 to pay for the shingles.

Have they budgeted correctly?  
Show all of your math thinking.



# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



# Roger's Roofing

## Analyzing Exemplars

Roger's Roofing Company was asked to put shingles on many new garages. The company decided to start with the smallest garage and continue until they were putting shingles on large business garages. The first garage took 3 loads of shingles. The second garage took 5 loads of shingles, and the third garage took 7 loads of shingles. The fourth garage took 9 loads of shingles. How many loads of shingles will the fifth garage use? How many loads of shingles will the fourteenth garage use? How many loads of shingles will the thirty-eighth garage use?



# Roger's Roofing

## Analyzing Exemplars

Each load of shingles cost \$120.00. The business that owns the nineteenth garage has budgeted \$5,500.00 to pay for the shingles. Have they budgeted correctly? Show all of your math thinking.



# Analyzing Student Work

- Examine each students' response.  
What does it suggest the student understands?  
Does not understand? Why?
  - Which response do you think shows the greatest understanding? Why?
  - Which response do you think shows the least understanding? Why?
- Rank your group's student responses from strongest to weakest using a 1, 2, 3, and 4 ranking. 1 being greatest understanding.



# Analyzing Student Work

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

# Sample A

I need to find out how many loads  
garage 5, 14, and 39 use. I need to find  
how much garage 19's load cost.  
I will make a chart and find a rule

Shingles

garage	Loads
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
14	29
15	31
16	33
17	35
18	37
19	39

The pattern is  $+2$ .

$$\begin{array}{r} \$120.00 \\ \times 39 \\ \hline 4,680.00 \end{array} \text{ enough}$$

garage	Loads
20	41
21	43
22	45
23	47
24	49
25	51
26	53
27	55
28	57
29	59
30	61
31	63
32	65
33	67
34	69
35	71
36	73
37	75
38	77

# Sample B

I need to find out how many loads garage 5, 14 and 19 use.  
I need to find out how much garage 19's load cost.  
I will make a chart and find a rule to complete  
my problem I'm trying to solve  
They have enough money because the shingles  
cost \$4,650 and they have \$5,000.00.

Garage	Load
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21
11	23
12	25
13	27
14	29
15	31
16	33
17	35
18	37
19	39

# Sample C

I need to find out how many loads garage 5, 14, 38, 19 use. I need to find how much garage 19's shingles cost. I will make a chart to find the rule.

Key  
L = load  
G = garage

Roger's Roofing

G	L
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

the pattern going down is +2

Look across

The student finds rule for solving for any number.

G	L
1	3
2	5
3	7

my Rule  
 $2 \times G + 1 = L$   
 or  
 $G \times 2 + 1 = L$

$$2 \times 14 + 1 = L \quad 14 \times 2 = 28$$

$$28 + 1 = L$$

$$29 = L \text{ for } 914$$

10/20

$$2 \times 38 + 1 = L \quad 38 \times 2 = 76$$

$$2 \times 19 + 1 = L$$

$$38 + 1 = 39$$

$$39 = L \text{ for } 939$$

$$939$$

$$\$120.00$$

$$\times 39$$

$$\begin{array}{r} \$414 \\ \$5,500.00 \\ - 4,680.00 \\ \hline \end{array}$$

$$\$4,680.60 \text{ they were rite}$$

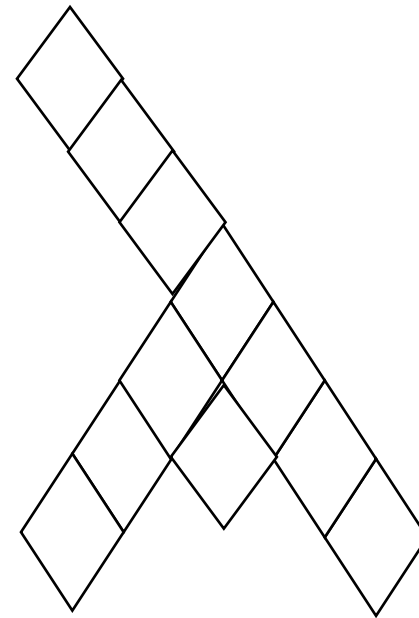
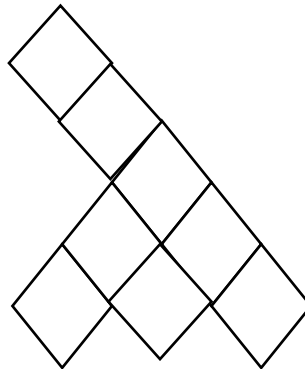
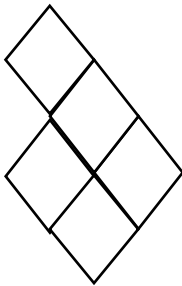
$$\begin{array}{r} \$820.00 \text{ left over} \end{array}$$

**Key**  
g- garage #  
L- loads

g - garage #  
L - loads

# Growing Rhombi

- How many rhombus pattern blocks are needed to make each figure? Complete the chart.



# Growing Rhombi

Figure Number	Number of Pattern Blocks
1	
2	
3	
4	
5	



# Growing Rhombi

- Using complete sentences, describe the patterns you see in the chart.
- How many rhombus pattern blocks would you need to make figures 7? 10? 15?
- When given any figure number, how can you find the number of rhombus pattern blocks needed to make that figure?

# Growing Rhombi

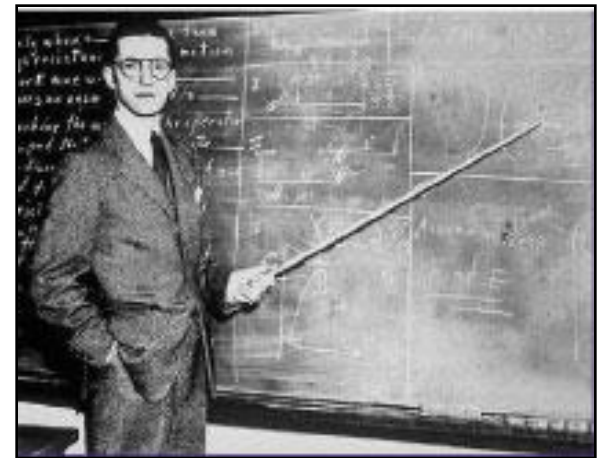
- What if a figure has 26 pattern blocks?  
What is its figure number?
- What type of pattern is formed by plotting the points on a grid?
- Could  $(12, 40)$  be a point on the graph?  
Explain your thinking.

# Growing Rhombi

- How does this problem relate to Roger's Roofing?
- How are they the same? Different?

# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



# U.S. Shirts

Using Tables, Graphs, and Equations

This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to find the total cost of customers' orders. The shop charges \$8 per shirt with a one-time set-up fee of \$15.

1. What is the total cost of an order for 10 shirts?
2. What is the total cost of an order for 100 shirts?
3. Explain how you found the total costs.
4. How many T-shirts can a customer buy for \$60?
5. How many T-shirts can a customer buy for \$250?
6. Explain how you found the number of shirts that can be purchased.



# U.S. Shirts

Using Tables, Graphs, and Equations

Make a table of values for the problem situation.

Labels	Number of Shirts Ordered	Total cost
	Shirts	\$
Unit		



# U.S. Shirts

Using Tables, Graphs, and Equations

- What are the variable quantities in this problem situation? Assign letters to represent these quantities and include each quantity's units.
- What are the constant quantities in this problem situation? Include the units that are used to measure these quantities.
- Which variable quantity depends on the other variable quantity?
- Which of the variables is the independent variable and which is the dependent variable?





# U.S. Shirts

Using Tables, Graphs, and Equations

- Create a graph of the data from your table. First, choose your bounds and intervals. Remember to label your graph clearly and add a title to the graph.
- Use your graph to determine the price of 40 shirts and 27 shirts. Use your graph to determine how many shirts can be purchased for \$300 and for \$540.



# U.S. Shirts

Using Tables, Graphs, and Equations

- Write an algebraic equation for the problem situation.
- In this lesson, you have represented the problem situation in four different ways: as a sentence, as a table, as a graph, and as an equation. Explain the advantages and disadvantages of each representation.



# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
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